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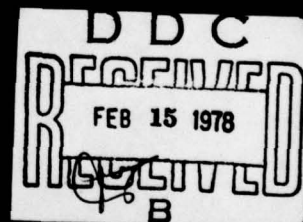
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A PROCEDURE FOR DETERMINING OPTIMAL SUBSIDIES AND
ECONOMIC ACTIVITY LEVELS IN AN ECONOMICALLY DEPRESSED AREA

by

TERJE HANSEN

Technical Report No. 132

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FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported, in part, by government contracts and industrial grants-in-aid. The work reported herein was supported (in part) by the U.S. Army Research Office under Contract DAAG29-76-C-0064.

Jeremy F. Shapiro
Acting Director

ABSTRACT

↘ In many developed countries there are areas with serious employment problems. Factor prices in such economically depressed areas are typically determined outside the area. Wage rates, for example, are given by nationwide bargaining by labor unions and are typically in excess of marginal productivity of labor in the depressed area. Economic activity in a depressed area is mainly undertaken by private entrepreneurs, who cannot operate without a subsidy of some kind.

A goal of public policy in the depressed area is full employment. This goal may be achieved either by expanding the public sector or by stimulating economic activity in the private sector through subsidies. The government seeks full employment at the lowest possible cost, the cost being the sum of total subsidies to the private sector and the net cost of the expanded public sector. The planning problem is formulated as a mixed integer programming problem. A solution procedure is suggested. The paper contains a constructed numerical example to illustrate the application of the solution procedure.

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A Procedure for Determining Optimal Subsidies and Economic Activity Levels in an Economically Depressed Area.

by

Terje Hansen

I. Introduction

Many developed countries are faced with the following economic planning problem:

1. There are areas of the country, in the subsequent called depressed areas, with serious employment problems. It is a national policy to maintain full employment.
2. Factor prices in a depressed area are typically determined outside the area. Wage rates, for example, are given by nationwide bargaining by labor unions and are typically in excess of marginal productivity of labor in the depressed area.
3. Economic activity in the depressed area is mainly undertaken by private entrepreneurs.

A certain amount of the labor force in the depressed area will be engaged in economic activities which are not subjected to external competition. Such activities include public activities and private basic economic activities in the depressed area. The remaining economic activities, either compete with products imported into the area or have to market their products outside the area. In either case, these activities

cannot be operated without a subsidy of some kind, if undertaken by private entrepreneurs.

A goal of public policy in the depressed area is full employment. This goal may be achieved either by expanding the public sector or by stimulating economic activity in the private sector through subsidies. The government seeks full employment at the lowest possible cost, the cost being the sum of total subsidies to the private sector and the net cost of the expanded public sector. The planning problem is formulated as a mixed integer programming problem. A solution procedure is suggested. In section 4 a numerical example is given.

2. The Economic Model

We shall consider the economy of a depressed area. There are n_1 possible activities that may be operated by the private sector and n_2 possible activities that may be operated by the public sector. Let

x_{1j} = the level of the j^{th} activity in the private sector,

x_{2k} = the level of the k^{th} activity in the public sector.

Prices, excluding subsidies, of all inputs and outputs are given. Consequently the net deficit of non-basic economic activities in the private sector are given. Likewise the net cost of public activities are given.

Let

d_{1j} = the net deficit of the j^{th} activity in the private sector,

d_{2k} = the net cost of the k^{th} activity in the public sector.

The objective of the government is to minimize¹

$$d_1' x_1 + d_2' x_2.$$

There is a goal of full employment in the depressed area in order to prevent depopulation. Let

a_{1ij} = amount of labor of type i required when the j^{th} activity in the private sector is operated at the unit level,

a_{2ik} = amount of labor of type i required when the k^{th} activity in the public sector is operated at the unit level,

a_i = amount of labor of type i not engaged in basic economic activities.

We thus get the full employment constraint

$$A_1 x_1 + A_2 x_2 = a.$$

¹ All vectors in this paper are column vectors. ' denotes transpose.

There are also a number of other considerations limiting the choice of economic activities. These are reflected by the constraints

$$B_1 x_1 + B_2 x_2 \geq b.$$

The government subsidizes factors of production and other inputs as well as outputs in the private sector. m different items are subsidized. For each item there is a maximum subsidy per unit in order to prevent misuse of public funds. Let

s_i = subsidy per unit of item i .

\bar{s}_i = maximum subsidy per unit of item i .

c_{ij} = amount of item i involved in the j^{th} activity in the private sector, when said activity is operated at the unit level.

z_{ij} = deficit, after subsidies have been added, of the j^{th} activity in the private sector, when said activity is operated at the unit level.

The cost of operating private activities include a reasonable return to the entrepreneur. We shall require that no private activity makes excess profits as a result of the subsidy system. Consequently we must have

$$\begin{aligned} C_1 s + z_1 &= d_1, \\ z_1 &\geq 0. \end{aligned}$$

Finally an activity in the private sector which is to be operated at a positive level must not make a deficit after subsidies have been added, i.e.

$$x_1' z_1 = 0.$$

3. A Procedure for Determining Optimal Subsidies and Economic Activity Levels.

The mathematical programming problem corresponding to the government's planning problem may now be stated:

$$\begin{aligned} 1.1 \quad & \text{minimize } d_1' x_1 + d_2' x_2, \\ & \text{subject to} \end{aligned}$$

$$1.2 \quad A_1 x_1 + A_2 x_2 = a,$$

$$1.3 \quad B_1 x_1 + B_2 x_2 \geq b,$$

$$1.4 \quad C_1 s + z_1 = d_1,$$

$$1.5 \quad z_1' x_1 = 0,$$

$$1.6 \quad x_1 \geq 0, x_2 \geq 0, \bar{s} \geq s \geq 0, z_1 \geq 0.$$

(1.1) - (1.6) is not a linear programming problem because of the presence of the constraint (1.5). By a suitable manipulation (1.1) - (1.6) may be transformed into a mixed integer programming problem. Let M be a large number and let y be a 0,1 vector. (1.5) may then be replaced by

$$\begin{aligned} (1.5') \quad & -M \cdot y + x_1 \leq 0, \\ & \frac{1}{M} z_1 + y \leq 1, \\ & 0 \leq y \leq 1. \end{aligned}$$

(1.1) - (1.4), (1.5'), (1.6) may then be solved as a mixed integer programming problem. Rather than using a standard integer programming routine, we shall suggest an algorithm which makes use of special features of the present problem.

Consider the set of vectors, $S(y)$, that satisfy the system of equations and inequalities

$$\begin{aligned} & \frac{1}{M} z_1 + y \leq 1, \\ & c_1 s + z_1 = d_1, \\ & 0 \leq s \leq \bar{s}, \quad z_1 \geq 0, \quad 0 \leq y \leq 1 \text{ and integer.} \end{aligned}$$

(1) may then be rewritten:

$$\text{minimize } d_1' x_1 + d_2' x_2 ,$$

subject to

$$A_1 x_1 + A_2 x_2 = a ,$$

$$(2) \quad B_1 x_1 + B_2 x_2 \geq b ,$$

$$-M y + x_1 \leq 0 ,$$

$$x_1 \geq 0, \quad x_2 \geq 0,$$

$$y \in S(y).$$

Let us define a subset $F(y)$ of $S(y)$, such that if $\bar{y} \in F(y)$, then there does not exist a $y \in S(y)$, different from \bar{y} , such that $y \geq \bar{y}$. Given the character of the constraints involving y , (2) is equivalent to solving

$$(3.1) \quad \text{minimize } d_1' x_1 + d_2' x_2 ,$$

subject to

$$(3.2) \quad A_1 x_1 + A_2 x_2 = a ,$$

$$(3.3) \quad B_1 x_1 + B_2 x_2 \geq b ,$$

$$(3.4) \quad -M y + x_1 \leq 0 ,$$

$$(3.5) \quad x_1 \geq 0, \quad x_2 \geq 0,$$

$$(3.6) \quad y \in F(y).$$

The strategy of our solution procedure is to generate the set $F(y)$ and next solve (3). Hopefully the number of elements in F are not very large, which may be a reasonable assumption if we have a problem of moderate size.

Consider the linear programming problem

$$\text{minimize} \quad y'z_1,$$

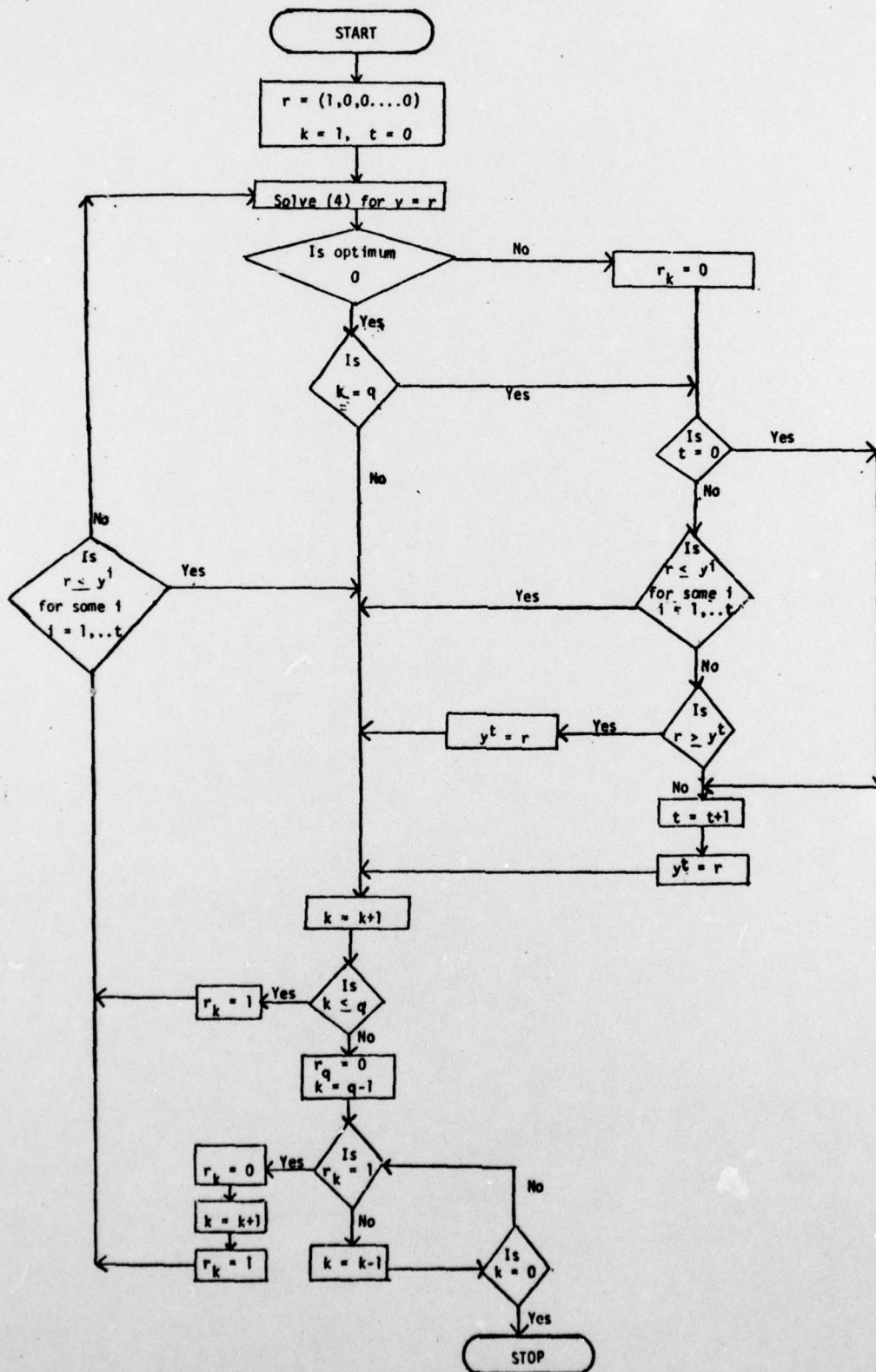
$$(4) \quad \text{subject to} \quad C_1 s + z_1 = d_1,$$

$$\bar{s} \geq s \geq 0, \quad z_1 \geq 0,$$

with y a given 0,1 vector. If and only if $y \in S(y)$ then the optimum of (4) is 0. Consequently we may test if $y \in S(y)$ by solving (4). Moreover if the optimum is strictly positive for $y = \bar{y}$, then it is clearly strictly positive for any $y \geq \bar{y}$. Consequently if \bar{y} is not a member of $S(y)$, neither is any $y \geq \bar{y}$. Specially consider the case $\hat{y} = (0, \dots, 1, \dots, 0)$, i.e. all components, but the j^{th} of \hat{y} are 0. If the optimum is strictly positive for $y = \hat{y}$, then no member of $S(y)$ has its j^{th} component positive.

In order to generate the set $F(y)$ we start by solving (4) for $y = (1, 0, 0, \dots, 0)$, $y = (0, 1, 0, \dots, 0)$ and $y = (0, 0, \dots, 0, 1)$. Suppose that the optimum of (4) is 0 for the first q vectors and positive for the remaining $n_1 - q$ vectors. Let $y^1 \dots y^{\ell}$ denote the vectors in $F(y)$. Let r be a 0,1 vector of the same dimension as y . The generation of the vectors in $F(y)$ is then illustrated by the flow chart on the next page.

Flow Chart Illustrating the Generation of the Set $F(y)$.



Preliminary experience with the algorithm that generates $F(y)$ indicates that the amount of computation is sensitive to the numbering of the q first activities. It appears that it is advantageous to number the activities such that

$$\text{Probability } (y_j^t > 0) < \text{Probability } (y_{j+1}^t > 0), \\ j = 1, \dots, q-1.$$

Consider a modified version of the linear programming problem given by (4)

$$\begin{aligned} &\text{minimize} \quad \sigma_j = \frac{1}{d_{1j}} z_{1j}, \\ (5) \quad &\text{subject to} \quad C_1 s + z_1 = d_1, \\ &\bar{s} \geq s \geq 0, \quad z_{1k} \geq 0, \quad k = 1, \dots, n_1, \quad k \neq j, \\ &z_{1j} \text{ unrestricted in sign.} \end{aligned}$$

Let $\hat{\sigma}_j$ denote the optimum to the above problem. For $j = 1, \dots, q$ we have $\hat{\sigma}_j \leq 0$ (if (5) also was solved for $j = q+1, \dots, n_1$ we would obviously have that $\hat{\sigma}_j > 0$). $\hat{\sigma}_j$ is in a sense a measure of the slack in the subsidy system with respect to the j^{th} activity. If $\hat{\sigma}_j$ is close to 0 the slack is small and one would consequently expect that the probability that a member of F shall have its j^{th} component positive is small. If on the other hand $\hat{\sigma}_j$ is very negative one would expect that the probability that a member of F shall have its j^{th} component positive is large. We consequently decided to renumber the first q activities such that

$$(> 0) \hat{\sigma}_j > \hat{\sigma}_{j+1}, \quad j = 1, \dots, q-1.$$

On the basis of limited computational experience it appears that this numbering procedure is relatively successful.

4. A Numerical Example

In order to illustrate the working of the algorithm presented in section 3 we shall consider a numerical example. The depressed area is assumed to consist of 2 subregions, one being relatively densely populated, region 1, the other being relatively sparsely populated, region 2. The private sector in the depressed area is mainly engaged in the catching and processing of fish. In region 2 these are the only activities. In region 1 shipyards may also be operated to service the fishing fleet. The possible private economic activities in the depressed area are as follows:

1. Fishing activity of type 1 in region 1
2. Fishing activity of type 2 in region 1
3. Fishing activity of type 3 in region 1
4. Filet of fish activity of type 1 in region 1
5. Filet of fish activity of type 2 in region 1
6. Canned fish activity in region 1
7. Salt fish activity in region 1
8. Shipyard activity in region 1
9. Fishing activity of type 1 in region 2
10. Fishing activity of type 2 in region 2
11. Fishing activity of type 3 in region 2
12. Filet of fish activity of type 2 in region 2
13. Canned fish activity in region 2
14. Saltfish activity in region 2

Activities 1-3 and 9-11 each produce one ton of fish per year when operated at the unit level. Activity 4-7 and 12-14 each require 1 ton of fish when operated at the unit level. Activity 8 produces 1 million kr of shipyard services per year when operated at the unit level.

The government operates an unemployment compensation program in the depressed area. There is also a program sponsoring the movement of families from region 2 to region 1. Because of the undesirability politically of having people unemployed or moved the cost of these activities are subjective and above actual cost. The government may locate public industry to region 1. The list of public activities are given below.

1. Unemployment compensation of male workers in region 1
2. Unemployment compensation of female workers in region 1
3. Unemployment compensation of male workers in region 2
4. Unemployment compensation of female workers in region 2
5. Moving of families from region 2 to 1
6. Public industry in region 1

There are 9 different items that are subsidized in the private sector (unit of measurement in parenthesis).

1. Male labor in region 1 (man year)
2. Female labor in region 1 (man year)
3. Male labor in region 2 (man year)
4. Female labor in region 2 (man year)

5. Capital used in region 1 (million N kr)
6. Capital used in region 2 (million N kr)
7. Fish caught (ton)
8. Fish input in the filet industry (ton)
9. Fish input in the canning industry (ton)

The data for our numerical example are given on the next pages.
Observe that we have assumed that $B_2 = 0$.

The Matrices A_1 and A_2 and the Vectors d_1 , d_2 .

Text	Private Sector														Public Sector						Labor Supply
	Activity														Activity						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	
<u>Region 1</u>																					
Male labor	120	60	30	25	50	25	25	100							100			-100	100	10,000	
Female labor	30	10	3	25	50	75	5	20							100			- 50	50	5,000	
<u>Region 2</u>																					
Male labor									120	60	30	50	25	25			100	100		20,000	
Female labor									30	10	3	50	75	5			100	50		10,000	
Deficit excluding subsidies of activities in the private sector	2.6	2.0	1.6	1.	2.5	2.5	1.	2.6	5.	3.2	2.	3.	3.	0.9							
Net cost of public activities															6.	4.	6.	4.	2.	5.	

The Matrix B_1 and the Vector b .

<u>Type of Constraint</u>	<u>b</u>	<u>Activity</u>													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Fish quota constraint	-400	-1	-1	-1						-1	-1	-1			
Fish processing constraint I	0	1	1	1	-1	-1	-1	-1		1	1	1	-1	-1	-1
Fish processing constraint II	0	-1	-1	-1	1	1	1	1		-1	-1	-1	1	1	1
Market constraint on filet products	-300				-1	-1							-1		
Market constraint on canned fish	-100						-1							-1	
Demand for shipyard services	0	3	2	1					-20	3	2	1			

The Matrix C_1 and the Vector \bar{s} .

The use of subsidized items by the various activities.									
Activity	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8	Item 9
1	120	30			40		1		
2	60	10			28		1		
3	30	3			20		1		
4	25	25			60			1	
5	50	50			40			1	
6	25	75			40				1
7	25	5			5				
8	100	20			80				
9			120	30		40	1		
10			60	10		28	1		
11			30	3		20	1		
12			50	50		40		1	
13			25	75		40			1
14			25	5		5			
Maximum subsidy per unit (million N kr)	0.03	0.02	0.03	0.02	0.05	0.05	2.0	1.0	1.0

We begin by solving (5) for $y = (1,0,0,\dots,0)$, $y = (0,1,0,\dots,0)$, ... $y = (0,0,0,\dots,1)$. The objective is strictly positive for the 1st, the 2nd, the 5th, the 7th and the 10th vector. Consequently all members of $F(y)$ will have their 1st, 2nd, 5th, 7th and 10th component 0.

After a suitable renumbering of the activities the algorithm described in the flow chart was applied to generate the members of $F(y)$. This involved the solution of 42 linear programming problems. Since the only difference between 2 successive programming problems is a minor adjustment of the objective we used the optimal solution of the former as an initial feasible basis for the latter linear programming problem. This way only a few pivot steps has to be performed for each linear programming problem. The generation of $F(y)$ required a total of 182 pivot steps.

The set $F(y)$ consists of only 4 vectors, which are given below.

Members of $F(y)$				
Component	y^1	y^2	y^3	y^4
1	0	0	0	0
2	0	0	0	0
3	1	1	0	0
4	1	1	1	1
5	0	0	0	0
6	0	1	1	1
7	0	0	0	0
8	1	1	1	0
9	1	1	1	1
10	0	0	0	0
11	1	1	0	1
12	1	0	1	1
13	1	1	1	1
14	0	0	1	0

We next solved (3.1) - (3.5) for $y = y^i$ ($i=1, \dots, 4$). For comparison we also give the solution to (3.1) - (3.5) for $y^0 = (1, 1, 1, \dots, 1)^4$.

The Solution to (3.1) - (3.5) for Alternative y 's

Activity	y^0	y^1	y^2	y^3	y^4
1	75.7	0	0	0	0
3	0	105.5	88.8	0	0
4	0	165.8	167.6	50.5	0
6	36.4	0	0	0	0
8	0	268.9	271.7	210.3	0
9	108.1	144.1	151.5	140.2	120
12	133.8	24.4	0	37.4	104
13	0	59.4	72.7	52.3	16
14	13.7	0	0	0	0
16	0	0	4.3	0	0
20	0	0	0	66.4	100
Value of Objective (million N kr)	1243	1377	1383	1408	1461

The optimal solution to our problem is consequently the solution we obtain for $y = y^1$. The minimum cost program, which has a cost of 1377 million N kr., involves 6 activities in the private sector and no public activities. There is a certain freedom with respect to the choice of the

subsidy system. The following subsidy vector

$$s = \begin{pmatrix} .023 \\ .02 \\ .02 \\ 0. \\ .02 \\ .008 \\ .85 \\ .49 \\ .12 \end{pmatrix},$$

satisfies the constraints of the problem 1.4-1.6.

If it had been feasible to subsidize the private activities on an individual basis (the case $y=y^0$) the cost would have amounted to 1243 million N kr., or some 10% less.

Our analysis could be extended to answer questions like

1. How will a change in the general subsidy system like an increase/decrease in maximum subsidies (\bar{s}) and addition of new items that are subsidized affect the optimal solution?
2. What is the marginal cost of the full employment policy?

These questions will not be examined in the present paper since our example is a constructed one. We hope, however, to return to questions such as these in an empirical application of our methodology.

From an algorithmic point of view we shall be interested in exploring how computation time varies with the dimension of the problem. This will be done in a future paper.